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Neutrino oscillations in non-inertial frames and the violation of the equivalence principle

Neutrino mixing induced by the equivalence principle violation

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Abstract. Neutrino oscillations are analyzed in an accelerating and rotating reference frame, assuming that the *gravitational* coupling of neutrinos is flavor dependent, which implies a violation of the equivalence principle. Unlike the usual studies in which a constant gravitational field is considered, such frames could represent a more suitable framework for testing if a breakdown of the equivalence principle occurs, due to the possibility to modulate the (simulated) gravitational field. The violation of the equivalence principle implies, for the case of a maximal gravitational mixing angle, the presence of an off-diagonal term in the mass matrix. The consequences on the evolution of flavor (mass) eigenstates of such a term are analyzed for solar (oscillations in the vacuum) and atmospheric neutrinos. We calculate the flavor oscillation probability in the non-inertial frame, which does depend on its angular velocity and linear acceleration, as well as on the energy of neutrinos, the mass-squared difference between two mass eigenstates, and on the measure of the degree of violation of the equivalence principle ($\Delta\gamma$). In particular, we find that the energy dependence disappears for vanishing mass-squared difference, unlike the result obtained by Gasperini, Halprin, Leung, and other physical mechanisms proposed as a viable explanation of neutrino oscillations. Estimations on the upper values of $\Delta\gamma$ are inferred for a rotating observer (with vanishing linear acceleration) comoving with the earth, hence $\omega \sim 7 \cdot 10^{-5}$ rad/sec, and all other alternative mechanisms generating the oscillation phenomena have been neglected. In this case we find that the constraints on $\Delta\gamma$ are given by $\Delta\gamma \leq 10^2$ for solar neutrinos and $\Delta\gamma \leq 10^6$ for atmospheric neutrinos.

1 Introduction

The possibility that neutrino particles could oscillate in different flavor states is one of the most discussed problems of today's theoretical and experimental physics.

Neutrino oscillations in the vacuum occur owing to the non-degeneration of the mass-matrix eigenvalues and to the difference of the mass eigenstates from weak interaction eigenstates. They have been introduced by Pontecorvo and Bilenky as a possible solution to the deficiency of the solar neutrino flux [1–3] and to the atmospheric neutrino problem [4]. An alternative solution to the observed solar neutrino deficit is given by the Mikheyev–Smirnov–Wolfenstein (MSW) mechanism [5], according to which neutrino oscillations are appreciably enhanced when crossing dense matter, as within the sun or other stars and in the early universe.

Another possibility to generate neutrino oscillations, one that will be used in this paper, is to suppose that neutrinos violate the equivalence principle through a different coupling of flavor eigenstates to gravity [6–8]. The main consequences of this approach are

(i) a linear dependence on the neutrino energy in the equation of evolution of the flavor states, which is different in comparison to the standard case, and
(ii) neutrino oscillations can occur even if neutrinos are massless particles.

Other alternative mechanisms have been proposed in the literature. We mention some of them: neutrino decay [9], Lorentz invariance [10], flavor changing neutral current [11], non-minimal couplings of neutrinos to a torsion field [12], and string inspired equivalence principle violation [13].

Very recently a lot of attention has been given to the physics of neutrinos propagating in curved backgrounds, in particular the effects of gravitational fields on the quantum mechanical phase of massive neutrinos [14–22]. It was shown in [14] that the gravitational correction to the neutrino phase is proportional to $G_{\rm N}\Delta m^2$, where $G_{\rm N}$ is the Newtonian gravitational constant and $\Delta m^2 = |m_2^2 - m_1^2|$ is the mass-squared difference. Nevertheless, in other ap-

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proaches, as for example in [15,16], such a term is cancelled. All these results have been carried out in the framework of general relativity. It is worth to note that extending the analysis of [15,16] to the Brans–Dicke theory of gravity, a term proportional to $G_{\rm N}\Delta m^2$ is recovered [17] and it vanishes in the limit in which the parameter characterizing the Brans–Dicke theory (ω) tends to infinity [23]. In this limit the Brans–Dicke theory coincides with general relativity in all its predictions [23,24].

On the other hand, experiments of recent years have led to convincing evidence for the existence of neutrino oscillations, with strong constraints of the theoretical models discussed above.

Such results have been found in different experiments:

(1) solar neutrino experiments [25–29],
(2) atmospheric neutrino experiments [30–34], and
(3) the accelerator LSND experiment [35]. Nevertheless, we have to note that many other neutrino oscillation experiments, with neutrinos produced by reactors and accelerators, *did not find any evidence* of neutrino oscillations.

The best fit in favor of neutrino oscillations are obtained for the following cases [36]:

(1) (MSW) small angle mixing region

$$\Delta m^2 \simeq (3 \div 10)\cdot 10^{-6}\,\mathrm{eV}^2, \tag{1}$$

$$\sin^2 2\theta_{\rm exp} \simeq (0.6 \div 1.3)\cdot 10^{-2};$$

(2) (MSW) large angle mixing region

$$\Delta m^2 \simeq (1 \div 20)\cdot 10^{-5}\,\mathrm{eV}^2, \quad \sin^2 2\theta_{\rm exp} \simeq 0.5 \div 0.9; \tag{2}$$

(3) solar vacuum oscillation

$$\Delta m^2 \simeq (0.5 \div 5)\cdot 10^{-10}\,\mathrm{eV}^2, \tag{3}$$

$$\sin^2 2\theta_{\rm exp} \simeq 0.67 \div 1;$$

(4) atmospheric neutrino oscillation (see also [37,38])

$$\Delta m^2 \simeq (10^{-3} \div 10^{-2})\,\mathrm{eV}^2, \quad \sin^2 2\theta_{\rm exp} \geq 0.8, \tag{4}$$

$$\Delta m^2 \simeq (0.5 \div 6)\cdot 10^{-3}\,\mathrm{eV}^2, \quad \sin^2 2\theta_{\rm exp} \geq 0.82;$$

(5) LSND experiment

$$\Delta m^2 \simeq (0.2 \div 10)\,\mathrm{eV}^2, \tag{5}$$

$$\sin^2 2\theta_{\rm exp} \simeq (0.2 \div 3)\cdot 10^{-2}.$$

$\theta_{\rm exp}$ is the *experimental* mixing angle. In the following, we will consider only the case of atmospheric and solar neutrino oscillations in the vacuum.

In this paper we analyze neutrino flavor oscillations in a non-inertial reference frame assuming the violation of the equivalence principle. Following [20] we derive the equation of evolution of neutrino states, which contains a non-vanishing *gravitational term* coming from the spinorial connections when calculated for the metric of a rotating and accelerating observer [39]. In order to have gravitationally induced flavor mixing we suppose a flavor dependence of the gravitational coupling, following the conjecture advanced by Gasperini and, independently, by Halprin and Leung. This opens a new scenario with respect to all previous papers [6–8,40,41] in which only effects due to the scalar gravitational field ϕ have been considered. Besides, the advantage to use non-inertial frames for testing a flavor non-diagonal coupling of neutrinos to *gravity*, is related to the possibility to *modulate the gravitational field*, simulated by the acceleration and angular velocity of the reference frame.

Constraints on the parameter characterizing the non-universal gravitational coupling with neutrino flavors, $\Delta\gamma$, are derived for observers comoving with the earth (with vanishing acceleration). Furthermore, we also assume that the mixing angle $\tilde{\theta}$ *measured* by such observers *is* the mixing angle measured in the experiments, $\tilde{\theta} \equiv w\theta_{\rm exp}$, and θ ($\theta_{\rm G}$), the usual mixing angle relating the flavor basis with the mass (gravitational) basis, is a free parameter. Finally, we neglect all other alternative mechanisms inducing the oscillation phenomena, as, in particular, the gravitational fields generated by the earth, confining ourselves to the consideration of only inertial effects and the violation of the equivalence principle.

The consequence of the violation of the equivalence principle on neutrino flavor mixing, when analyzed in non-inertial frames, is the appearence of an off-diagonal term in the flavor mass-matrix which is proportional to $\Delta\gamma\boldsymbol{\omega}\cdot\boldsymbol{p}$, $\boldsymbol{\omega}$ being the angular velocity of the observer and $\boldsymbol{p}$ the momentum of the neutrino. It is this contribution to the oscillations that gives rise to new effects which could provide a peculiar signature for probing if a violation of the equivalence principle occurs. In particular we find the following.

(i) For vanishing mass-squared difference, $\Delta m^2 = 0$, and for maximal gravitational mixing angle $\theta_{\rm G} = \pi/4$, we define an *effective* mass-squared difference $\Delta m^2_{\rm eff}$ which is proportional to $\sim \Delta\gamma\omega E$, where $E \sim p$ is the neutrino energy. Determining $\Delta\gamma$, whose upper limit turns out to be of the order 10^2 for solar neutrinos (with energy $\sim 20\,\mathrm{MeV}$), and 10^6 for atmospheric neutrinos (with energy $\sim 10 \div 10^2\,\mathrm{GeV}$), we infer $\Delta m^2_{\rm eff} \sim 10^{-10}\,\mathrm{eV}^2$ for the former, and $\Delta m^2_{\rm eff} \sim 10^{-3} \div 10^{-2}\,\mathrm{eV}^2$ for the latter. Both values of $\Delta m^2_{\rm eff}$ are in agreement with the best fit of the experimental data.
(ii) The flavor mixing probability is energy independent when $\Delta m^2 = 0$. This implies a different behavior in comparison to the other alternative mechanisms [6–10]. It must be noted that an energy-independent probability is also derived in [11,12].

The organization of this paper is as follows. In Sect. 2 we briefly discuss the Dirac equation in curved space-time and calculate the probability that neutrino flavor oscillations occur with respect to an accelerating and rotating observer. In Sect. 3 we discuss the phenomenological consequences of inertial effects on the solar neutrino problem. Section 4 is devoted an analysis of the inertial effects on solar neutrinos produced by interactions of the cosmic rays with the solar atmosphere. In Sect. 5 we apply such an analysis to atmospheric neutrinos. Conclusions are drawn in Sect. 6.

2 Neutrino oscillations in an accelerating and rotating frame

As in [20], the generalized neutrino phase is given by (hereafter we use the natural units $\hbar = c = 1$)

$$|\psi_f(\lambda)\rangle = \sum_j U_{fj} \mathrm{e}^{\mathrm{i}\int_{\lambda_0}^{\lambda} P\cdot p_{\mathrm{null}} \mathrm{d}\lambda'} |\nu_j\rangle, \tag{6}$$

where f is the flavor index and j the mass one. U_{fj} are the matrix elements transforming flavor and mass bases, P is the four-momentum operator generating space-time translations of the eigenstates and $p^{\mu}_{\mathrm{null}} = \mathrm{d}x^{\mu}/\mathrm{d}\lambda$ is the tangent vector to the neutrino worldline x^{μ}, parameterized by λ. The covariant Dirac equation in curved space-time [42] is

$$[\mathrm{i}\gamma^{\mu}(x)D_{\mu} - m]\psi = 0,$$

where the matrices $\gamma^{\mu}(x)$ are related to the usual Dirac matrices $\gamma^{\hat{a}}$ by means of the vierbein fields $e^{\hat{a}}_{\mu}(x)$, where the Greek (Latin with hat) indices refer to curved (flat) space-time. D_{μ} is defined as $D_{\mu} = \partial_{\mu} + \Gamma_{\mu}(x)$, where ∂_{μ} is the usual derivative and $\Gamma_{\mu}(x)$ is the spinorial connection defined by

$$\Gamma_{\mu}(x) = \frac{1}{8}[\gamma^{\hat{a}}, \gamma^{\hat{b}}] e^{\nu}_{\hat{a}} e_{\nu\hat{b};\mu}$$

(a semicolon represents the covariant derivative). The relations

$$\gamma^{\hat{a}}[\gamma^{\hat{b}}, \gamma^{\hat{c}}] = 2\eta^{\hat{a}\hat{b}}\gamma^{\hat{c}} - 2\eta^{\hat{a}\hat{c}}\gamma^{\hat{b}} - 2\mathrm{i}\varepsilon^{\hat{d}\hat{a}\hat{b}\hat{c}}\gamma^5\gamma^{\hat{d}},$$

where $\eta^{\hat{a}\hat{b}}$ is the metric tensor in flat spacetime, $\varepsilon^{\hat{d}\hat{a}\hat{b}\hat{c}}$ is the totally antisymmetric tensor, $\gamma^5 = \mathrm{i}\gamma^{\hat{0}}\gamma^{\hat{1}}\gamma^{\hat{2}}\gamma^{\hat{3}}$, and $\{\gamma^5, \gamma^{\hat{a}}\} = 0$, allow one to recast the non-vanishing contribution from the spin connection in the form

$$\gamma^{\hat{a}} e^{\mu}_{\hat{a}} \Gamma_{\mu} = \gamma^{\hat{a}} e^{\mu}_{\hat{a}} \left\{ \mathrm{i}A_{\mathrm{G}\mu} \left[-(-g)^{-1/2}\frac{\gamma^5}{2} \right] \right\}, \tag{7}$$

where

$$A^{\mu}_{\mathrm{G}} = \frac{1}{4}\sqrt{-g}\, e^{\mu}_{\hat{a}} \varepsilon^{\hat{d}\hat{a}\hat{b}\hat{c}} (e_{\hat{b}\nu,\sigma} - e_{\hat{b}\sigma,\nu}) e^{\nu}_{\hat{c}} e^{\sigma}_{\hat{d}}, \tag{8}$$

and $g \equiv \det(g_{\mu\nu})$. $g_{\mu\nu}$ is the metric tensor of curved space-time. The momentum operator P_{μ}, used to calculate the phase of neutrino oscillations, is derived from the mass shell condition

$$(P_{\mu} + A_{\mathrm{G}\mu}\mathcal{P}_{\mathrm{L}})(P^{\mu} + A^{\mu}_{\mathrm{G}}\mathcal{P}_{\mathrm{L}}) = -M^2_f, \tag{9}$$

where $\mathcal{P}_{\mathrm{L}}$ is the left-handed projection operator and M^2_f is the vacuum mass matrix in the flavor base

$$M^2_f = U \begin{pmatrix} m_1^2 & 0 \\ 0 & m_2^2 \end{pmatrix} U^{\dagger}, \; U = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}. \tag{10}$$

θ is the vacuum mixing angle. Ignoring terms of the order $\mathcal{O}(A^2_{\mathrm{G}})$ and $\mathcal{O}(A_{\mathrm{G}}M_f)$, one gets that, for relativistic neutrinos moving along generic trajectories parameterized by λ, the column vector of flavor amplitude

$$\chi(\lambda) = \begin{pmatrix} \langle\nu_e|\psi(\lambda)\rangle \\ \langle\nu_{\mu}|\psi(\lambda)\rangle \end{pmatrix} \tag{11}$$

satisfies the equation

$$\mathrm{i}\frac{\mathrm{d}\chi}{\mathrm{d}\lambda} = \left(\frac{M^2_f}{2} + p\cdot A_{\mathrm{G}}\mathcal{P}_{\mathrm{L}} \right) \chi. \tag{12}$$

In deriving (12), one uses the relation $P^0 = p^0$ and $P^i \approx p^i$ [20]. In an accelerating and rotating frame, the vierbein fields $e^{\hat{a}}_{\mu}(x)$ are given by [39]

$$e^{\hat{0}}_0 = 1 + \boldsymbol{a}\cdot\boldsymbol{x}, \quad e^{\hat{0}}_m = 0, \quad e^{\hat{k}}_0 = \varepsilon^{\hat{k}\hat{l}\hat{m}}\omega^{\hat{l}}x^{\hat{m}}, \quad e^{\hat{k}}_l = \delta^k_l, \tag{13}$$

where $k, l, m = 1, 2, 3$, $x^{\mu} = (x^0, \boldsymbol{x})$ are the local coordinates for the observer at the origin and $\boldsymbol{a}$, $\boldsymbol{\omega}$ are the acceleration and angular velocity of the frame, respectively. The components $e^{\mu}_{\hat{a}}(x)$ and $e_{\mu\hat{a}}(x)$ are calculated by using the metric tensors $g_{\mu\nu}$ and $\eta_{\hat{a}\hat{b}}$, with $g_{\mu\nu}$ given by the line element [39]

$$\begin{aligned} \mathrm{d}s^2 &= [(1+\boldsymbol{a}\cdot\boldsymbol{x})^2 + (\boldsymbol{\omega}\cdot\boldsymbol{x})^2 - (\boldsymbol{\omega}\cdot\boldsymbol{\omega})(\boldsymbol{x}\cdot\boldsymbol{x})](\mathrm{d}x^0)^2 \\ &\quad - 2\mathrm{d}x^0\mathrm{d}\boldsymbol{x}\cdot(\boldsymbol{\omega}\wedge\boldsymbol{x}) - \mathrm{d}\boldsymbol{x}\cdot\mathrm{d}\boldsymbol{x}. \end{aligned} \tag{14}$$

Inserting (13) into (8), one gets the components of A^{μ}_{G}:

$$A^0_{\mathrm{G}} = 0, \quad \boldsymbol{A}_{\mathrm{G}} = \frac{\sqrt{-g}}{2}\frac{1}{1+\boldsymbol{a}\cdot\boldsymbol{x}}\{2\boldsymbol{\omega} - [\boldsymbol{a}\wedge(\boldsymbol{x}\wedge\boldsymbol{\omega})]\}. \tag{15}$$

Notice in (12) that the amount of neutrino mixing depends on the direction of the neutrino momentum with respect to the angular velocity and acceleration of the reference frame.

The gravitational term A^{μ}_{G} in (12) does not give any contribution to the flavor mixing (it is flavor diagonal). The last one, in fact, occurs only as a consequence of a breakdown of the universality of the gravitational coupling of neutrinos with different flavors. This assumption allows one to re-write (12) in the form [40]

$$\begin{aligned} \mathrm{i}\frac{\mathrm{d}}{\mathrm{d}\lambda}\begin{pmatrix} a_e \\ a_{\mu} \end{pmatrix} &= \left[\frac{\Delta}{2}\begin{pmatrix} -\cos 2\theta & \sin 2\theta \\ \sin 2\theta & \cos 2\theta \end{pmatrix} \right. \\ &\quad \left. + \Delta\gamma\boldsymbol{p}\cdot\boldsymbol{A}_{\mathrm{G}}\begin{pmatrix} -\cos 2\theta_{\mathrm{G}} & \sin 2\theta_{\mathrm{G}} \\ \sin 2\theta_{\mathrm{G}} & \cos 2\theta_{\mathrm{G}} \end{pmatrix} \right] \begin{pmatrix} a_e \\ a_{\mu} \end{pmatrix}, \end{aligned} \tag{16}$$

where $a_f \equiv \langle\nu_f|\psi(\lambda)\rangle$, $f = e, \mu$, and $\Delta\gamma = \gamma_1 - \gamma_2$ is a measure of the equivalence principle violation, γ_1 and γ_2 being the two different couplings of the neutrino flavors to A_{G}. In (16) $\Delta \equiv \Delta m^2/2$. The term $\Delta\gamma\boldsymbol{p}\cdot\boldsymbol{A}_{\mathrm{G}}$ in (16) is the analogue of the term $\Delta\gamma E|\phi|$ for neutrinos propagating in a gravitational field ϕ. For the sake of simplicity, we consider the case of a maximal gravitational mixing angle, i.e. $\theta_G = \pi/4$. Then (16) becomes

$$\mathrm{i}\frac{\mathrm{d}}{\mathrm{d}\lambda}\begin{pmatrix} a_e \\ a_\mu \end{pmatrix} = \mathcal{T}\begin{pmatrix} a_e \\ a_\mu \end{pmatrix}, \tag{17}$$

where the matrix $\mathcal{T}$ is defined by

$$\mathcal{T} = \begin{bmatrix} -\frac{\Delta}{2}\cos 2\theta & \frac{\Delta}{2}\sin 2\theta + \Delta\chi \\ \frac{\Delta}{2}\sin 2\theta + \Delta\chi & \frac{\Delta}{2}\cos 2\theta \end{bmatrix} \tag{18}$$

up to the $(m_1^2 + m_2^2)/2$ term, proportional to the identity matrix. Here $\Delta\chi \equiv \Delta\gamma \boldsymbol{p}\cdot\boldsymbol{A}_G$. We restrict to the flavors e, μ, but this analysis works also for different neutrino flavors. To determine the mass eigenstates $|\nu_1\rangle$ and $|\nu_2\rangle$, corresponding to a fixed value of the acceleration and angular velocity of the frame, one has to diagonalize the matrix $\mathcal{T}$. Using the standard procedure, one writes the mass eigenstates as a superposition of flavor eigenstates

$$\begin{aligned} |\nu_1(\lambda)\rangle &= \cos\tilde{\theta}(\lambda)|\nu_e\rangle - \sin\tilde{\theta}(\lambda)|\nu_\mu\rangle, \\ |\nu_2(\lambda)\rangle &= \sin\tilde{\theta}(\lambda)|\nu_e\rangle + \cos\tilde{\theta}(\lambda)|\nu_\mu\rangle, \end{aligned} \tag{19}$$

where the mixing angle $\tilde{\theta}$ is defined in terms of the vacuum mixing angle

$$\tan 2\tilde{\theta} = \frac{\Delta\sin 2\theta + 2\Delta\gamma\boldsymbol{p}\cdot\boldsymbol{A}_G}{\Delta\cos 2\theta}. \tag{20}$$

We note that $\tilde{\theta} \to \theta$ as $\boldsymbol{A}_G \to 0$ (i.e. $\boldsymbol{a} \to 0, \boldsymbol{\omega} \to 0$). The corresponding eigenvalues are

$$\tau_{1,2} = \pm\sqrt{\frac{\Delta^2}{4}\cos^2 2\theta + \left[\frac{\Delta}{2}\sin 2\theta + (\Delta\gamma\boldsymbol{p}\cdot\boldsymbol{A}_G)\right]^2}. \tag{21}$$

Writing $|\psi(\lambda)\rangle = a_1(\lambda)|\nu_1\rangle + a_2(\lambda)|\nu_2\rangle$, (17) assumes the form

$$\mathrm{i}\frac{\mathrm{d}}{\mathrm{d}\lambda}\begin{pmatrix} a_1 \\ a_2 \end{pmatrix} = \begin{pmatrix} \tau_1 & 0 \\ 0 & \tau_2 \end{pmatrix}\begin{pmatrix} a_1 \\ a_2 \end{pmatrix}, \tag{22}$$

where $a_k = \langle\nu_k|\psi(\lambda)\rangle, k = 1, 2$, and

$$\begin{pmatrix} a_1 \\ a_2 \end{pmatrix} = \tilde{U}\begin{pmatrix} a_e \\ a_\mu \end{pmatrix}, \quad \tilde{U} = \begin{pmatrix} \cos\tilde{\theta} & \sin\tilde{\theta} \\ -\sin\tilde{\theta} & \cos\tilde{\theta} \end{pmatrix}. \tag{23}$$

We have used the condition $\mathrm{d}\tilde{\theta}/\mathrm{d}\lambda \approx 0$ in order that (22) is a diagonal matrix. This means that we are neglecting the variations of acceleration and angular velocity, with respect to the affine parameter λ, in comparing to their magnitudes. Equation (22) implies

$$a_k(\lambda) = a_k(0)\exp[-\mathrm{i}\alpha_k(\lambda)], \tag{24}$$

$$\alpha_k(\lambda) \equiv \int_{\lambda_0}^{\lambda} \tau_k \mathrm{d}\lambda', \quad k = 1, 2.$$

For the initial condition $|\psi(0)\rangle = |\nu_e\rangle$, the state $|\psi(\lambda)\rangle$ is

$$\begin{aligned} |\psi(\lambda)\rangle &= [\cos^2\tilde{\theta}\mathrm{e}^{\mathrm{i}\alpha} + \sin^2\tilde{\theta}\mathrm{e}^{-\mathrm{i}\alpha}]|\nu_e\rangle \\ &\quad + [-\cos\tilde{\theta}\sin\tilde{\theta}\mathrm{e}^{\mathrm{i}\alpha} + \sin\tilde{\theta}\cos\tilde{\theta}\mathrm{e}^{-\mathrm{i}\alpha}]|\nu_\mu\rangle, \end{aligned} \tag{25}$$

where $\alpha = \alpha_1 = -\alpha_2$. The probability to observe an electronic neutrino is therefore

$$P_{\nu_e\to\nu_e} \equiv |\langle\nu_e|\psi(\lambda)\rangle|^2 = 1 - \sin^2 2\tilde{\theta}\sin^2\alpha. \tag{26}$$

In the next sections we will discuss the particular case of rotating reference frames in order to estimate the contributions to neutrino oscillations when inertial effects and the equivalence principle violation are taken into account.

3 Inertial effects on solar neutrinos

Consequences of inertial effects on neutrino oscillations can be derived from (20). We consider the case of a vanishing linear acceleration, $\boldsymbol{a} = (0, 0, 0)$, so that the reference frame is only rotating. Then (20) reads

$$\tan 2\tilde{\theta} = \tan 2\theta + \frac{4\Delta\gamma\boldsymbol{\omega}\cdot\boldsymbol{p}}{\Delta m^2\cos 2\theta}. \tag{27}$$

Before starting with our analysis, it is worth to spend some words on the main sources of solar neutrinos. The main emission of electron neutrinos is due to the reactions [43]

$$\begin{aligned} p + p &\to d + e^+ + \nu_e, & E &\leq 0.42\,\mathrm{MeV}, \\ e^- + {}^7\mathrm{Be} &\to {}^7\mathrm{Li} + \nu_e, & E &\sim 0.86\,\mathrm{MeV}, \\ {}^8\mathrm{B} &\to {}^8\mathrm{Be} + e^+ + \nu_e, & E &\leq 20\,\mathrm{MeV}. \end{aligned}$$

Neutrinos produced by these reactions are called pp, ^{7}Be, and ^{8}B neutrinos, respectively. The first ones give the major contribution to the flux, the second ones contribute about 10%, and, finally, the ^{8}B neutrinos constitute a very small part of the total flux. Nevertheless, due to the high threshold energy in experiments, the ^{8}B neutrinos with energy of the order 20 MeV give the major contribution to the event rates. For example, in the Homestake experiment with threshold energy $E \sim 0.81$ MeV, pp neutrinos cannot be detected, and ^{8}B and ^{7}Be neutrinos contribute with 77% and 15% to the event rate, respectively. Thus, in the following, we will confine ourselves to ^{8}B neutrinos.

In (27), the mixing angle $\tilde{\theta}$ measured by the rotating observer is interpreted as the experimental mixing angle θ_{exp}. θ then is a free parameter (let us remember that we have put $\theta_G = \pi/4$). In this sense, the rotating observer is comoving with the earth so that ω is given by $\omega \sim 7\cdot 10^{-5}$ rad/sec. Taking into account the angle $\beta = 66.5°$ of the direction of the angular velocity $\boldsymbol{\omega}$ and the direction of the neutrinos coming from the Sun[1], (27) can be recast in the form

$$\tan 2\tilde{\theta} = \tan 2\theta + \Delta\gamma\frac{1.4\cdot 10^{-12}\,\mathrm{eV}^2}{\Delta m^2\cos 2\theta}, \tag{28}$$

[1] The rotation axes of the earth is inclined by an angle of 23.5° with respect to the normal of the elliptic plane, so that $\beta = 90° - 23.5° = 66.5°$

where the ultra-relativistic condition $p \sim E \sim 20\,\mathrm{MeV}$ has been used.

For the value $\sin^2 2\theta_{\mathrm{exp}} = 1$ ($\theta_{\mathrm{exp}} = \tilde{\theta} = \pi/4$) one gets $\tan 2\tilde{\theta} \to \infty$. We have two possibilities:
(1) $\Delta m^2 \neq 0$ and $\theta = \pi/4$. The vacuum mixing angle measured by the non-inertial observer does coincide with the one measured by the inertial observer, $\tilde{\theta} = \theta = \pi/4$. The probability that electron neutrinos preserve their flavor is, from (26),

$$P_{\nu_e \to \nu_e} = \cos^2\left(\pi \frac{\Delta l}{\lambda_1}\right), \tag{29}$$

where Δl is the physical distance between the source and the detection point of the neutrinos and λ_1 is the characteristic oscillation length given by

$$\lambda_1 = \frac{\pi}{\dfrac{\Delta m^2}{4E} + \Delta\gamma\omega\cos\beta}. \tag{30}$$

Assuming $\lambda_1 \sim d_{\mathrm{SE}} \sim 1.5 \cdot 10^{11}\,\mathrm{m}$ (d_{SE} is the distance sun–earth), for $\Delta m^2 \sim 5 \cdot 10^{-10}\,\mathrm{eV}^2$ and $\Delta m^2 \sim 5 \cdot 10^{-11}\,\mathrm{eV}^2$, we get $\Delta\gamma \sim -90$ ($\gamma_2 > \gamma_1$) and $\Delta\gamma \sim 2 \cdot 10^2$ ($\gamma_1 > \gamma_2$), respectively.
(2) $\theta \neq \pi/4$ and $\Delta m^2 = 0$. In this case either the mass-matrix is degenerate or the neutrinos are massless particles. Neutrino oscillations occur as a *pure* inertial effect and as a consequence of the equivalence principle violation. In fact, from the equation of evolution (17), we infer

$$\mathrm{i}\frac{\mathrm{d}}{\mathrm{d}\lambda}\begin{pmatrix} a_e \\ a_\mu \end{pmatrix} = \Delta\gamma\boldsymbol{\omega}\cdot\boldsymbol{p}\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}\begin{pmatrix} a_e \\ a_\mu \end{pmatrix}. \tag{31}$$

The flavor transition probability (26) becomes

$$P_{\nu_e \to \nu_e} = \cos^2(\Delta\gamma\omega\cos\beta\Delta l). \tag{32}$$

The dependence on the energy of the neutrinos disappears and the probability depends only on the angular velocity of the rotating observer and on $\Delta\gamma$. This behavior differs in a substantial way from alternative mechanisms, in particular from the equivalence principle violation for neutrinos propagating in a gravitational field ϕ, in which the dependence on the energy has the functional form $\Delta\gamma\phi E\Delta l$. The constraint on $\Delta\gamma$ is now calculated geometrically, i.e. independent on the energy and on the mass-squared differences of the neutrinos, from the characteristic oscillation length coming from (32):

$$\lambda_2 = \frac{\pi}{|\Delta\gamma|\omega\cos\beta} \tag{33}$$

$$= 1.5\,\mathrm{km}\left(\frac{10^{-10}\,\mathrm{eV}}{\Delta\gamma\omega}\right)$$

$$= 2.3\,\mathrm{km}\left(\frac{10^{6}\,\mathrm{rad/sec}}{\Delta\gamma\omega}\right). \tag{34}$$

The numerical value of $\lambda_2 \sim d_{\mathrm{SE}} \sim 1.5 \cdot 10^{11}\,\mathrm{m}$ leads again to the value $|\Delta\gamma| \sim 10^2$.

The previous results can be read from a different point of view: Inertial effects *simulate* neutrino flavor oscillations as induced by an effective non-zero mass-squared difference $\Delta m^2_{\mathrm{eff}}$. The last one is derived by equating (33) to the characteristic oscillation length derived in the standard description of neutrino oscillations (i.e. $4\pi E/\Delta m^2_{\mathrm{eff}}$). It then follows that

$$\Delta m^2_{\mathrm{eff}} = 4|\Delta\gamma|E\omega\cos\beta. \tag{35}$$

For solar neutrinos, (35) implies

$$\Delta m^2_{\mathrm{eff}} \approx 10^{-10}\,\mathrm{eV}^2, \tag{36}$$

which is in agreement with the best fit of the experimental data.

Let us discuss, finally, the particular case of vanishing vacuum mixing angle θ, $\theta = 0$. Equation (28) reduces to the form

$$\tan 2\tilde{\theta} = \Delta\gamma\frac{1.45 \cdot 10^{-12}\,\mathrm{eV}^2}{\Delta m^2}. \tag{37}$$

In Table 1, the values of $\Delta\gamma$ corresponding to different experimental values of Δm^2 and $\sin^2 2\tilde{\theta}$ are reported. Values of the mass-squared difference $\Delta m^2 \sim 5 \cdot 10^{-11} \div 5 \cdot 10^{-10}\,\mathrm{eV}^2$ and $\sin^2 2\tilde{\theta} \sim 0.67$ yield the range of variability for $|\Delta\gamma|$ given by $\Delta\gamma \sim 10 \div 10^2$.

Table 1. Estimations of $\Delta\gamma$ for given mass-squared difference Δm^2 and mixing angle $\sin^2 2\tilde{\theta}$

Δm^2 (eV^2)	$\sin^2 2\tilde{\theta}$	$\Delta\gamma <$
$5 \cdot 10^{-11}$	0.67	5
$5 \cdot 10^{-10}$	0.67	50
10^{-9}	0.67	$9 \cdot 10^2$

The survival probability (26) becomes

$$P_{\nu_e \to \nu_e} = 1 - A\sin^2\left(\pi\frac{\Delta l}{\lambda_3}\right), \tag{38}$$

where

$$A = \frac{(4\Delta\gamma\omega E\cos\beta)^2}{(\Delta m^2)^2 + (4\Delta\gamma\omega E\cos\beta)^2}, \tag{39}$$

and

$$\lambda_3 = \frac{4\pi E}{\sqrt{(\Delta m^2)^2 + (4\Delta\gamma\omega E\cos\beta)^2}}. \tag{40}$$

λ_3 is the oscillation length. In particular, the probability that electron neutrinos oscillate in muon neutrinos, for $\Delta\gamma \sim 50$, $\Delta m^2 \sim 5 \cdot 10^{-10}\,\mathrm{eV}^2$, and $E \sim 20\,\mathrm{MeV}$, is $P_{\nu_e \to \nu_\mu} \sim 0.02$.

4 Solar neutrinos generated by cosmic rays

In the previous section we have considered solar neutrinos emitted by thermonuclear processes, in agreement to the standard solar theory. In this section, we will focus our attention to solar neutrinos produced by interactions of cosmic rays with the solar atmosphere [44]. The effect of the interactions is to produce particles which propagate through the sun. They can either decay or give rise to secondary interactions which produce other particles, generating in such a way a cascade of particles. Neutrinos are produced by the decay of particles, mainly muons, in the cascade. This scenario is similar to what happens in the terrestrial atmosphere, when cosmic rays interact with it. However, the solar atmosphere is less dense at the typical interaction heights, so that a larger fraction of mesons will decay producing in the sun a huge amount of neutrinos in comparison to the ones produced at the earth. Electron and muon neutrinos produced through this process have an energy of the order of or greater than 10 GeV [44]. The analysis of previous section allows one to get, for such neutrinos, the following estimates ($E\sim 10\,\mathrm{GeV}$). When $\theta_{\rm exp}=\tilde\theta=\pi/4$, one has two possibilities:

(1) For $\Delta m^2\neq 0$ and $\theta=\pi/4$, (30) allows one to determine the value of $\Delta\gamma$. In fact, by putting $\lambda_1\sim d_{\rm SE}\sim 1.5\cdot 10^{11}$m and $\Delta m^2\sim(0.5\div 5)\cdot 10^{-10}\,\mathrm{eV}^2$, it follows that $\Delta\gamma\sim 2\cdot 10^2$.

(2) $\Delta m^2=0$ and $\theta\neq\pi/4$. We have introduced an effective mass-squared difference, (35), related to the angular velocity of the observer and the energy of neutrinos. Specifying (35) for solar neutrinos generated by cosmic rays, we get

$$\Delta m^2_{\rm eff}=4\Delta\gamma E\omega\cos\beta=8\Delta\gamma\cdot 10^{-10}\,\mathrm{eV}^2. \qquad (41)$$

Since the geometrical estimation of $\Delta\gamma$ gives a value of the order 10^2, see (33), (41) implies $\Delta m^2_{\rm eff}\sim 10^{-8}\,\mathrm{eV}^2$.

Finally, we envisage the case $\theta=0$. Equation (27) reads

$$\tan 2\tilde\theta=\Delta\gamma\frac{4E\omega\cos\beta}{\Delta m^2}\sim\Delta\gamma\frac{7.4\cdot 10^{-10}\,\mathrm{eV}^2}{\Delta m^2}. \qquad (42)$$

For $\Delta m^2\sim 5\cdot 10^{-11}\div 5\cdot 10^{-10}\,\mathrm{eV}^2$ and $\sin^2 2\tilde\theta\sim 0.67$, (42) allows us to infer the constraint $\Delta\gamma\sim 0.01\div 0.1$. Furthermore, one can calculate the flavor mixing probability from (26). For $\Delta\gamma\sim 0.1$, $\Delta m^2\sim 5\cdot 10^{-10}\,\mathrm{eV}^2$, and $E\sim 10\div 10^2\,\mathrm{G\,eV}$, (38) gives the probability $P_{\nu_e\to\nu_\mu}\sim 0\div 0.06$, i.e. it increases with energy.

5 Inertial effects on atmospheric neutrinos

In this section, the previous analysis is extended to the case of atmospheric neutrinos. We consider neutrinos crossing the earth (so that the oscillation length is $\sim 12.6\cdot 10^3\,\mathrm{km}$) with energies of the order $10\div 10^2\,\mathrm{GeV}$, and we put the angle between the angular velocity and the direction of the neutrinos equal to zero. We also neglect the neutrino interaction with matter background.

For the value $\sin^2 2\theta_{\rm exp}=1$ ($\theta_{\rm exp}=\tilde\theta=\pi/4$) one gets $\tan 2\tilde\theta\to\infty$. We have, as already said, two possibilities:

(1) $\Delta m^2\neq 0$ and $\theta=\pi/4$. From (30), putting $\lambda_1\sim 12.6\cdot 10^3\,\mathrm{km}$, we derive the order of magnitude of $\Delta\gamma$ reported in Table 2. We get $|\Delta\gamma|\sim 10^5\div 10^6$. Notice the variation of the sign of $\Delta\gamma$ in passing from $\Delta m^2\sim 10^{-3}\,\mathrm{eV}^2$ to $\Delta m^2\sim 10^{-2}\,\mathrm{eV}^2$ at $E\sim 10\,\mathrm{GeV}$.

Table 2. Estimations of $\Delta\gamma$ for given energies E and mass-squared difference Δm^2

E (GeV)	Δm^2 (eV^2)	$\lvert\Delta\gamma\rvert <$
10	10^{-3}	$6\cdot 10^5$
10	10^{-2}	$4\cdot 10^6$
10^2	10^{-3}	10^6
10^2	10^{-2}	$6\cdot 10^5$

(2) $\theta\neq\pi/4$ and $\Delta m^2=0$. Assuming $\lambda_2\sim 12.6\cdot 10^3\,\mathrm{km}$ ($\beta=0$) in (33), one geometrically derives $\Delta\gamma\sim 10^6$. By using this value, (35) gives

$$\Delta m^2_{\rm eff}\sim 10^{-3}\div 10^{-2}\,\mathrm{eV}^2,$$

in agreement with the best fit of the experimental data.

As final case to investigate, we put $\theta=0$. For the value $\sin^2 2\tilde\theta\sim 0.8$, (28) reduces to the form

$$\tan 2\tilde\theta=\Delta\gamma\frac{4\omega E}{\Delta m^2}\sim 2,$$

from which $\Delta\gamma\sim 10^5$, which is in agreement with the above estimations. From (38), the maximum of the flavor mixing probability for energies $E\sim 10\div 10^2\,\mathrm{GeV}$ is $P_{\nu_e\to\nu_\mu}=0.12$ when $\Delta m^2\sim 10^{-3}\,\mathrm{eV}^2$, and $P_{\nu_e\to\nu_\mu}=0.35$ when $\Delta m^2\sim 10^{-2}\,\mathrm{eV}^2$.

6 Conclusions

The problem of neutrino oscillations has been widely discussed during last years and, though a lot of theories have been proposed, a definitive solution is still far away. The main ideas which try to explain the solar neutrino deficit are based on the vacuum neutrino oscillations and on the MSW mechanism.

Very recently, attention has been paid to the study of the effects of gravitational fields and of the violation of the equivalence principle on the quantum mechanical phase of mixed states, in particular on neutrino oscillations. In this paper we have faced this problem by considering the neutrino flavor mixing in an accelerating and rotating reference frame.

Inertial effects and the request of the violation of the equivalence principle induce an additional off-diagonal term in the flavor mass-matrix, which allows one to write down a relation between the mixing angles, $\tilde\theta$, θ and $\theta_{\rm G}(=\pi/4)$, and the mass-squared difference Δm^2, (20). This equation has several implications that we have analyzed in a rotating reference frame, with vanishing linear acceleration. Such a frame is comoving with the earth, so that $\omega\sim 7\cdot 10^{-5}\,\mathrm{rad/sec}$, and the mixing angle $\tilde\theta$ is the one measured in experiments, $\theta_{\rm exp}$.

By putting $\Delta m^2=0$, we have found that the oscillations are *modulated* only by the angular velocity of the observer and the difference of couplings $\Delta\gamma$. One can introduce an effective mass-squared difference $\Delta m^2_{\rm eff}$ which simulates the neutrino oscillations effect as induced by non-vanishing neutrino masses. For solar neutrinos we geometrically have derived $\Delta\gamma \sim 10^2$ (see (30)), which allows us to determine the effective mass-squared difference $\Delta m^2_{\rm eff} \sim 10^{-10}\,{\rm eV}^2$.

Similar results hold for atmospheric neutrinos. In this case $\Delta\gamma \sim 10^6$, and the effective mass-squared difference is of the order $\Delta m^2_{\rm eff} \sim 10^{-3} \div 10^{-2}\,{\rm eV}^2$.

We have also analyzed the solar neutrinos produced by interactions of cosmic rays with the solar atmosphere. The energy of such neutrinos is of the order of (or greater than) 10 GeV. $\Delta\gamma$ is $\sim 10^2$, and the corresponding effective mass-squared difference is $\Delta m^2_{\rm eff} \sim 10^{-8}\,{\rm eV}^2$. When $\theta = 0$, we have obtained the constraint $\Delta\gamma \sim 10^{-2}$.

The constraint on $\Delta\gamma$ estimated for solar and atmospheric neutrinos differs considerably from the one obtained by considering neutrinos propagating in a gravitational field. In fact, for solar neutrinos one has

$$0.65 \leq \sin^2 2\theta_{\rm G} \leq 1, \quad 3\cdot 10^{-20} < |\Delta\gamma| < 3\cdot 10^{-18}, \quad (43)$$

obtained for a gravitational field $|\phi| \sim 10^{-5}$ [40,45]. Such a discrepancy would suggest to rule out the equivalence principle violation mechanism as a possible solution to the solar and atmospheric neutrino problem. Nevertheless, some comments are in order. First, it must be noted that, as argued in [19,40], physical results depend on the choice of the metric when the breakdown of the equivalence principle occurs. Second, in deriving (43), the static metric has a diagonal form (Schwarzschild-like), unlike the one of the rotating observer where non-diagonal terms are present, leading to different constraints on $\Delta\gamma$, as we have seen. We again stress that estimates of $\Delta\gamma$ have been inferred by assuming inertial effects (with the equivalence principle violation) as the only one responsible for oscillations. Of course, taking into account other alternative mechanisms, in particular ones induced by the gravitational field of the earth, inertial effects would give a non-dominant contribution, at least for frames comoving with the earth. A simple inspection of (16) shows in fact that inertial effects could play a relevant role only by requiring high frequency rotating reference frames.

Let us finally compare the previous results with recent ones in which the relevance of the alternative models has been investigated. The oscillation lengths derived in the previous sections are here reported:

$$\lambda_1^{-1} = \frac{\Delta m^2}{4\pi E} + \frac{\Delta\gamma\omega\cos\psi}{\pi}, \quad (44)$$

$$\lambda_2^{-1} = \frac{\Delta\gamma\omega\cos\psi}{\pi}, \quad (45)$$

$$\lambda_3^{-1} = \frac{\sqrt{(\Delta m^2)^2 + (4\Delta\gamma\omega\cos\psi)^2}}{4\pi E}. \quad (46)$$

$\psi = \beta = 66.5°$ for solar neutrinos, and $\psi = 0$ for atmospheric neutrinos. The λ_i, $i = 1,2,3$, show a differentw dependence on the energy. As is well known, in the cases of interest the oscillation length λ depends on the energy as $\lambda^{-1} \sim E^n$. Hence, λ_1^{-1} corresponds to a standard oscillation plus the equivalence principle violation in non-inertial frames, $(n = -1) \oplus (n = 0)$; λ_2^{-1} corresponds to the equivalence principle violation in non-inertial frames, $n = 0$; andw finally, λ_3^{-1} presents a non-trivial dependence on the energy, which reduces to the standard oscillation formula $\Delta m^2/4\pi E$ for $\Delta\gamma = 0$ or $\omega = 0$, and to λ_2^{-2} for vanishing mass-squared difference.

The behavior $\lambda^{-1} \sim E$ coming from a flavor violating gravitational field (as proposed in [6–8]) appeared to fit the SuperKamiokande data, as well as the other alternative mechanisms [9,11,46–48]. A different analysis of such data, including for example upward-going muon events, has been performed in [49]. In these papers, it is shown that the best fit does confirm, at least for atmospheric neutrinos, the standard scenario as the dominant oscillation mechanism, whereas the alternative mechanisms do not provide a viable description of the data. In this analysis, the authors consider

(i) the equivalence principle violation for neutrino propagating in a gravitational field,
(ii) neutrino decay, and
(iii) the flavor changing neutral currents mechanism in which the flavor mixing probability is energy independent.

In view of these results, in particular the point (iii), one could conclude that inertial effects together with the violation of the equivalence principle seem to be disfavored for atmospheric neutrinos, or at most, they are subleading processes with small amplitude which coexist with the leading, large amplitude of the standard oscillations.

Nevertheless, the above conclusions are not definitive and much more studies are still necessary for understanding the neutrino physics and whether inertial effects could provide, as suggested in this paper, further tests for probing the validity of the equivalence principle violation. Moreover, a complete analysis should also involve a non-vanishing linear acceleration, terms in the mass matrix for generic values of $\theta_{\rm G}$, the matter background effects, the extension to three and four neutrino flavors, that have been neglected here, which could lead, among other things, to more stringent limits on $\Delta\gamma$. Such issues will be addressed in the future.

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